Warm-up!

A) Let $f(x) = 60$. Write an equivalent expression in the form .

$60\left(\frac{1}{3}\right)^{-2x}\left(\frac{1}{3}\right)^{1}$

$20\left(\frac{1}{3}\right)^{-2x} = 20\left(\left(\frac{1}{3}\right)^{-2}\right)^{x}$

$20(9)^{x}$

B) A jacket is normally \$80 but it is on sale for off! What is the price of the jacket on the day of the sale?

$\$80 - 20\% \cdot \$80$

$\$80 - \$16$

$\$64$

$80\% \cdot \$80$

$\frac{4}{5} \cdot 80$

64

C) What does "half life" mean in chemistry/physics?

Exponential Functions Day 4:

- (2.5)
- (2.5) Exponential growth/decay with percentage increase/decrease
- (2.4+2.5) Changing units

Homework: Exponential Functions in Context

Quiz Next Class: Day 1-3

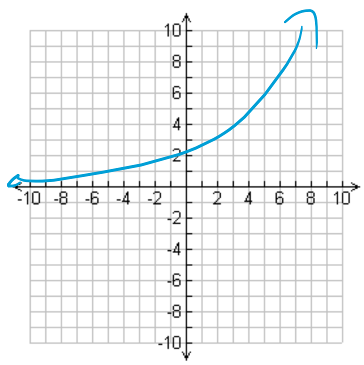
In many applications, the most convenient base is the irrational number $e = 2.71828182845 \ldots$

You should memorize this number to at least 20 decimal places (just kidding. Memorize it to…zero decimal places)

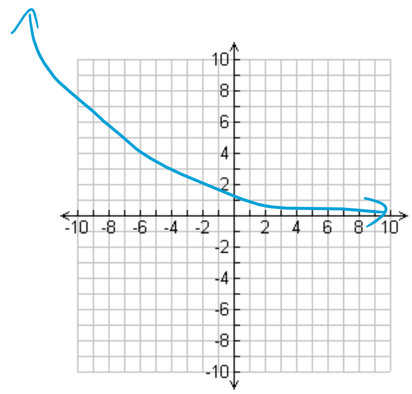
Let's briefly visit Desmos to make sure we know how to do e^{-3} and $e^{2.5}$

To graph:

$f(x) = e^x$



$g(x) = e^{-x}$



Jack bought some magic beans and planted them. A huge beanstalk started growing. Every day, the height of the beanstalk is $\frac{3}{2}$ times the height it was the day before.

Good strategy for all questions today: Come up with two points from the given information. Test them with the function you create.

A) If the beanstalk is 20 feet tall today, write a function that models how tall the beanstalk will be t days from now.

$g_1 = 20 \cdot \left(\frac{3}{2}\right)^1 = 30 \qquad g_2 = 45 \rightarrow R = \frac{3}{2}$

$g_n = g_1 R^{n-1}$

$g_n = 30\left(\frac{3}{2}\right)^{n-1}$

$g(t) = 30\left(\frac{3}{2}\right)^{t-1}$

$30\left(\frac{3}{2}\right)^t \left(\frac{3}{2}\right)^{-1}$

$30 \cdot \left(\frac{3}{2}\right)^t \cdot \frac{2}{3}$

$20\left(\frac{3}{2}\right)^t$

$g_n = g_0 R^n$

$g(t) = 20\left(\frac{3}{2}\right)^t$

B) By what percentage is the height of the beanstalk increasing every day?

20×1.5

$20 \times (1 + 0.5) \rightarrow 50\%$

Each year solar panels seem to get cheaper. In fact, the general trend seems to be that each year solar panels cost only 0.89 times the price from the previous year. In 2016, installing solar panels cost about \$3 per watt.

A) Write a function that models the cost of solar panels t years after 2016.

$g_0 = 3$ $\quad g_1 = 3 \cdot .89$ $\quad g_n = g_0 R^n$

$f(t) = 3(.89)^t$ $\quad g_n = 3(.89)^n$

B) By what percent does the cost of solar panels go down each year?

$\times\ 0.89$

$\times (1 - 0.11)$

The US Federal Reserve targets a 2% inflation rate. This means that they aim for prices to generally increase by 2% each year. The price of a burrito bowl at Chipotle is currently $\$11$. Assuming the price increases by 2% each year, write a function that predicts the price t years from now.

$$g_0 = 11 \xrightarrow{\times R = 1.02} g_1 =$$

$$g_n = g_0 \, R^{n-0}$$

$$g_n = 11\,(1.02)^n$$

$$f(t) = 11\,(1.02)^t$$

Each year, fossil fuels play a less and less important role in generating energy in the United Kingdom. In 2023, $100\ \mathrm{TWh}$ of energy in the UK came from fossil fuels. Assume that each year 20% less energy comes from fossil fuels. Write a function that models how much energy can be expected to be generated by fossil fuels in the UK t years after 2023.

$(100\% - 20\%$

$g_0 = 2023 \qquad g_1 =$

$\times R = 0.8$

$g_n = g_0 R^n$

$g_n = 100(0.8)^n$

$f(t) = 100(0.8)^t$

Dubnium-270 has a half life of 1 day. This means the amount of Dubnium-270 in a sample decreases by 50% every 1 day. A sample initially has 10 picograms of Dubnium-270.

$g_0 = 10$

$g_1 = 5$

$g_2 = 2.5$

$g_3 = 1.25$

A) Write a function modeling how much Dubnium-270 remains in the sample after t days.

$f(t) = 10(0.5)^t$

$f(1)$

$f(2)$

B) Write a function modeling how much Dubnium-270 remains in the sample after t hours.

$h(t) = 10(0.5)^{t/24}$

$h(24)$

$h(48)$

C) Write a function modeling how much Dubnium-270 remains in the sample after t weeks.

$W(t) = 10(0.5)^{7t}$

$w(1) = f(7)$

$\cancel{10(0.5)^{t/7}}$

Sally opens a savings account with a great interest rate at her bank. Every quarter of a year (3 months), the amount of money in Sally's account increases by 1%. She begins with $\$500$

$g_0 = 500$

$g_1 = 500 \times 1.01$

$g_2 = 500 \times 1.01 \times 1.01$

A) Write a function modelling how much money will be in Sally's account after time t if t is measured in quarters of a year.

$q(t) = 500(1.01)^t$

$q(4)$

B) Write a function modelling how much money will be in Sally's account after time t if t is measured in years.

$y(t) = 500(1.01)^{4t}$

$y(1)$

C) Write a function modelling how much money will be in Sally's account after time t if t is measured in months.

$m(t) = 500(1.01)^{t/3}$

$m(12)$

A pan of baked orzo is removed from the oven. Initially, the pan is $250°C$ hotter than the temperature in the kitchen. But every ten minutes, the difference between the pan's temperature and the room's temperature decreases by 85%.

A) Write a function that models the difference in temperature t minutes after the pan is removed from the oven.

$$m(t) = 250(0.15)^{t/10}$$

B) Write a function that models the difference in temperature t hours after the pan is removed from the oven.

$$h(t) = 250(0.15)^{6t}$$

The global average internet speed seems to increase by 80% every three years. In 2022, the global average internet speed was approximately 100 megabits per second.

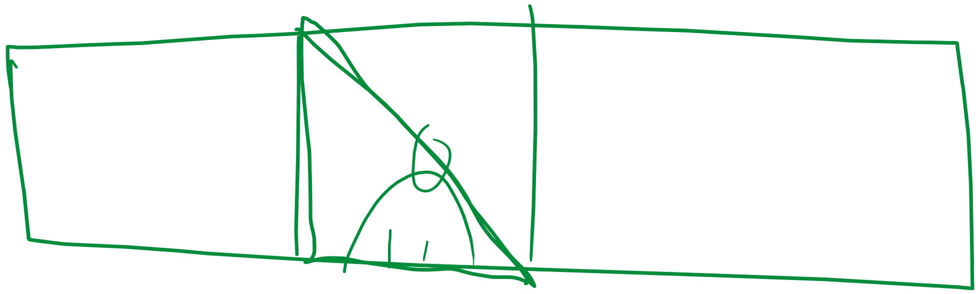
A) Write a function that models the global average internet speed t years after 2022.

$$f(t) = 100(1.8)^{t/3}$$

B) Use a calculator/Desmos and your function from part A to predict the global average internet speed in the year 2000.

$$f(-22) = 1.343$$

Speed in 2025: 180



2.7182818284590452

Jan 1	2025	\$100	
			+\$25
Apr 1	2025	\$125	
			+ \$31.25
July 1	2025	\$156.25	
			+\$39.06
Sep 1	2025	\$195.31	
			+\$49.08
Jan 1	2026	\$271.82	